Malaysian Journal of Mathematical Sciences 15(1): 109–123 (2021)

MALAYSIAN JOURNAL OF MATHEMATICAL SCIENCES

Journal homepage: http://einspem.upm.edu.my/journal

Contributions to Generalized Derivation on Prime Near-Ring with its Application in the Prime Graph

Khan, M. A.*

Department of Mathematics and Computer Science, Umaru Musa Yar'adua University, Katsina, Nigeria

E-mail: moharram.alikhan@umyu.edu.ng
* *Corresponding author*

Received: 22 December 2019
Accepted: 20 September 2020

ABSTRACT

In this paper, we discuss the notion of prime near- ring, which was introduced by Bell and Mason (1987) and Wang (1994) independently. Recently, many authors have investigated commutativity of prime and semi-prime rings admitting suitably constrained derivations (see Daif and Bell (1992), Gölbaşi and Koç (2009) for references). Daif and Bell (1992) showed that a prime ring R must be commutative if it admits a derivation d such that either $d([x, y]) = [x, y]$ or $d([x, y]) = -[x, y]$ $\forall\ x, y \in I$, where I is a non-zero ideal of R. Beck (1988) linked a commutative ring R to a graph by using the elements R as vertices and any two vertices x, y are adjacent if and only if $x \neq y$ and $xy = 0$. The zero-divisor graph of a commutative ring R is a graph with the set of non-zero zero-divisors of R as the vertices and any two vertices x, y are adjacent if and only if $x \neq y$ and $xy = 0$. Some comparable results on near-rings have also been derived Beidar et al. (1996), Boua and Oukhtite (2011), Gölbaşi and Koç (2009). The prime graph of a near-ring is a graph with vertices as the set of elements of N and edges as the set of vertex pair $\{x, y\}$ such that $xNy = 0$. Indeed N is prime if and only if prime graph is a star

graph (see Bhavanari et al. (2010)). The objective of the present paper is to extend some results on prime rings admitting generalized derivation to prime near-rings, and some results on relationship between the prime graph and the zero-divisor graph of N. In addition, examples are given to demonstrate the primeness in the hypothesis is not superfluous. Finally, we pose some open problems.

Keywords: Commutativity, generalized derivation, prime graph, prime near-ring, star graph, zero-divisor graph.

1. Introduction

We mean by N a right near ring if $(N,+)$ is an additive group, $(N,\cdot)$ a semi group and $(x+y)\cdot z = x\cdot z + y\cdot z$, $\forall\ x,y,z \in N$. Recently, **?** proved new commutativity theorems for rings and applications of commutativity theorems for rings to near-rings. In addition, some polynomial identities with exponents depending on x and y for a certain class of near-rings were established. **?** and **?** investigate some new results on commutativity of prime near-rings and 3-prime near-rings, respectively. A right near-ring N is a prime if, for all $x, y \in N$ such that $xNy = \{0\}$ implies $x = 0$ or $y = 0$ (see Bell and Mason (1987) for details), and the symbols $[x,y] = xy - yx$ (resp. $x \circ y = xy + yx$) represent commutator (resp. anti commutator). An additive map $d : N \longrightarrow N$ is called a derivation if $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$ or (equivalently, as noted in Wang (1994), that is, $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$). An additive map $G : N \longrightarrow N$ is said to be a generalized derivation on N if there exists a derivation $d : N \longrightarrow N$ such that $G(xy) = G(x)y + xd(y)$ holds for all $x, y \in N$. Assume that $C(N)$ is the multiplicative center of N (i.e. $C(N) = \{x \in N \mid xy = yx\ \forall\ y \in N\}$. Recall $C(N) \neq \phi$, as $0 \in C(N)$.

The prime graph of a near-ring N is a graph with elements of N as the vertices and any two vertices $x, y \in N$ are adjacent if and only if $xNy = \{0\}$ or $yNx = \{0\}$. If N is a commutative ring then the zero-divisor graph of N is a subgroup of the prime graph N. The idea of a zero-divisor graph was introduced by Beck (1988) and modified by Anderson and Naseer (1993). Let R be a commutative ring and $Z(R)$ be the set of non-zero zero-divisors of R. The zero-divisor graph, as defined in Anderson and Livingston (1999), is the graph with vertices $Z(R)$ and edges $a - b$ if and only if $ab = 0$ and $a = b$. Later these ideas were applied to noncommutative rings by Redmond (2004). If the requirement that the near-ring have identity is removed, then other zero-divisor graphs on three vertices can be realized.

For example, in Robinson and Foulds (1980), two noncommutative rings (without multiplicative identity) were found to have the zero-divisor graphs. Examples of graphs of rings on three vertices without unity can be seen in Redmond (2004). For an introduction to graph theory and near-rings including basic definitions and properties (see references Redmond (2004) and Groenewald (1991) for details). Daif and Bell (1992) proved that the non-zero ideal A of a semi prime ring R is contained in the center of R if

$$d([x,y]) = [x,y] \text{ or } d([x,y]) = -[x,y] \text{ for all } x,y \in A. \tag{1}$$

From the literature, a number of authors have studied commutativity theorems for prime or semi prime rings admitting derivation d or generalized derivation G satisfying the condition (1). For example, **?** extend some commutativity results for derivations to left generalized derivations in prime rings. In this continuation, one can investigate some algebraic as well as differential identities to establish the commutativity of a near-ring N admitting generalized derivations. Now, we define the following near-ring properties to establish the commutativity of a prime near-ring N if N admits a generalized derivation G together with one of the conditions:

(p) For every $x, y \in N$ there exist polynomials with integral coefficients $f(x), g(x) \in Z[x]$ such that G satisfies (i) $G[x,y] = f(x)(x \circ y)g(x)$ or (ii) $G[x,y] = -f(x)(x \circ y)g(x)$.

(p_1) For every $x, y \in N$ there exist polynomials with integral coefficients $f(x), g(x) \in Z[x]$ such that G satisfies (iii) $G(x \circ y) = f(x)[x,y]g(x)$ or (iv) $G(x \circ y) = -f(x)[x,y]g(x)$.

(p_2) For every $x, y \in N$ there exist polynomials with integral coefficients $f(x), g(x) \in Z[x]$ such that G satisfies (v) $G[x,y] = f(y)(x \circ y)g(y)$ or (vi) $G[x,y] = -f(y)(x \circ y)g(y)$.

(p_3) For every $x, y \in N$ there exist polynomials with integral coefficients $f(x), g(x) \in Z[x]$ such that G satisfies (vii) $G(x \circ y) = f(y)[x,y]g(y)$ or (viii) $G(x \circ y) = -f(y)[x,y]g(y)$.

In Section 2, we study the commutativity of a prime near-ring N admitting a non-zero derivation G under the above properties (p), (p_1), (p_2) and (p_3). As a consequence of the results in this section, we extend Theorem due to Daif and Bell (1992). Section 3 is devoted to related results on prime graph. The results of this section are analogue to results in Section 2. The aim of this paper is to continue this line of investigation and to discuss the commutativity of prime near-rings with prime graph involving generalized derivation in more general setting. In addition we present some open problems for future research work.

2. The Results

We begin our investigation with some well-known facts and results in near-rings which will be used frequently throughout the text.

Fact 2.1

(i) Set, for any $x, y \in N, [x, y] = xy - yx$ and $xoy = xy + yx$. then yx in place of y in the above identities, we find that $[x, yx] = xyx - yxx = [x, y]x$; $[xy, y] = [x, y]y$; $(xyoy) = xyy + yxy = (xoy)y$ *and* $(xoyx) = (xoy)x$.

(ii) Take

$$f(x) = \sum_{i=0}^{i=m} x^i, \; g(x) = \sum_{j=0}^{j=n} x^j, \; f(y) = \sum_{r=0}^{r=p} y^r, \text{ and } g(y) = \sum_{s=0}^{s=q} x^s.$$

Then

$$xf(x) = \sum_{i=0}^{i=m} x^{i+1} = f(x)x; \; xg(x) = \sum_{j=0}^{j=n} x^{j=1} = g(x)x;$$

$$yf(y) = \sum_{r=0}^{r=p} x^{r+1} = f(y)y; \text{ and } yg(y) = \sum_{s=0}^{s=n} x^{s+1} = g(y)y.$$

Fact 2.2 [Bell and Mason (1987), Theorem 2]. Let N be a prime near-ring. If N admits a non-zero derivation d for which $d(N) \subseteq Z(N)$, $Z(N)$ is the center of N, and then N is a commutative ring.

Fact 2.3 Bhavanari et al. (2010). Assume that prime graph of N is a graph with vertices as the set of elements of N and edges as the set of elements of N and edges as the set of vertex pair $\{x, y\}$ such that $xNy = \{0\}$ or $yNx = \{0\}$. Then N is prime if and only if the prime graph of a near-ring N is a star graph.

Fact 2.4 Bhavanari et al. (2010). The zero-divisor graph of a commutative ring R is a graph with the set of non-zero divisors of R as the vertices any two vertices x, y are adjacent if and only if $x \neq y$ and $xy = 0$.

In the following theorems, we prove the main results of Khan and Madugu (2017) involving generalized derivations in more setting.

Theorem 2.1. If a prime near-ring N admits a generalized derivation G associated with a non-zero derivation d satisfies the property (p), then N is a commutative ring.

Theorem 2.2. If a prime near-ring N admits a generalized derivation G associated with a non-zero derivation d satisfies the property (p_1), then N is a commutative ring.

Theorem 2.3. If a prime near-ring N admits a generalized derivation G associated with a non-zero derivation d satisfies the property (p_2) then N is a commutative ring.

Theorem 2.4. If a prime near-ring N admits a generalized derivation G associated with a non-zero derivation d satisfies the property (p_3) then N is a commutative ring.

Proof of Theorem 2.1

Assume the condition (i) of (p) as follows

$$G[x, y] = f(x)(x \circ y)g(x) \text{ for all } x, y \in N. \tag{2}$$

Replacing y by yx in (2), we get

$$G[x, yx] = f(x)(x \circ yx)g(x). \tag{3}$$

By definition of generalized derivation, we have

$$G([x, y]x) = G([x, y])x + [x, y]d(x). \tag{4}$$

Substituting the values of (2) and (3) in (4) and using the Fact 2.1 (ii), we get

$$\sum_{i=0}^{i=m} x^i(x \circ y) \sum_{j=0}^{j=n} x^{j+1} = \sum_{i=0}^{i=m} x^i(x \circ y) \sum_{j=0}^{j=n} x^{j+1} + [x, y]d(x).$$

This implies that
$$[x, y]d(x) = 0. \tag{5}$$
Replacing y by zy in (5), we find that
$$[x, zy]d(x) = [x, z]yd(x) = 0 \ \ \forall x, y, z \in N.$$
This yields
$$[x, z]Nd(x) = \{0\} \ \ \forall x, z \in N. \tag{6}$$
By an application of primeness of N, the relation (6) gives that for each $x \in N$,
$$d(x) = 0 \ \text{ or } \ x \in Z(N). \tag{7}$$
Assume that $x \in Z(N)$ then
$$xy = yx, \ \text{ for all } \ y \in N. \tag{8}$$
Taking derivation both sides in (8), we have $d(xy) = d(yx)$.

This implies that $d(x)y + xd(y) = d(y)x + yd(x)$, but $x \in Z(N)$, then $d(x)y = yd(x)$, for all $y \in N$ or $d(x) \in Z(N)$. Hence, (7) implies that for all $x \in Z(N)$, $d(x) \in Z(N)$, that is, $d(N) \subseteq Z(N)$. In view of Fact 2.2, we obtain N is a commutative ring.

Now, if condition (ii) of (p) holds, then putting yx for y in (ii), we can similarly find that
$$G([x, y]x) = G([x, yx]) = -\sum_{i=0}^{i=m} x^i(x \circ y) \sum_{j=0}^{j=m} x^{j+1} \tag{9}$$
$$= -\sum_{i=0}^{i=m} x^i(x \circ y) \sum_{j=0}^{j=m} x^{j+1}. \tag{10}$$
By definition of generalized derivation, we have $G([x, y]x) = G([x, y])x + [x, y]d(x)$.

Combining (9) and property (ii), we obtain
$$-\sum_{i=0}^{i=m} x^i(x \circ y) \sum_{j=0}^{j=m} x^{j+1} = -\sum_{i=0}^{i=m} x^i(x \circ y) \sum_{j=0}^{j=m} x^{j+1} + [x, y]d(x).$$
This implies that $[x, y]d(x) = 0$. Use the same argument as above to get the requited result.

Proof of Theorem 2.2. Assume the condition (iii) of (p_1) as follows

$$G(xoy) = f(x)[x,y]g(x). \tag{11}$$

Replacing y by yx in (11), we get

$$G(xoyx) = G(xoy)x = f(x)[x,y]xg(x), \forall x,y \in N. \tag{12}$$

By definition of generalized derivation, we have

$$G((xoy)x) = G(xoy)x + (xoy)d(x). \tag{13}$$

Putting the values of (11) and (12) in (13), we find that

$$\sum_{i=0}^{i=m} x^i[x,y] \sum_{j=0}^{j=m} x^{j+1} = \sum_{i=0}^{i=m} x^i[x,y] \sum_{j=0}^{j=m} x^{j+1} + (xoy)d(x).$$

This implies that

$$(xoy)d(x) = 0 \forall x,y \in N \tag{14}$$

or

$$xyd(x) = -yxd(x) \forall x,y \in N.$$

Setting y by zy in (14), we obtain

$$xzyd(x) = -zyxd(x) = (-z)(-xyd(x)) = (-z)(-x)yd(x), \forall x,y,z \in N.$$

This gives

$$(xz - (-z)(-x))yd(x) = 0, \forall x,y,z \in N. \tag{15}$$

Putting x by $-x$ in (15), we get $(-xz + zx)yd(-x) = 0$, which implies

$$-[z,x]yd(x) = [z,x]yd(-x) = 0, \forall x,y,z \in N. \tag{16}$$

By an application of primeness of N, the relation (16) gives that for each $x \in N$,

$$d(x) = 0 \textit{ or } x \in Z(N). \tag{17}$$

The relation (17) is same as (7), using the procedures as in the proof of Theorem 2.1, we find that N is a commutative ring.

Next, assume condition (iv) of (p_1) holds, putting yx for y in (iv), we can similarly get

$$G((xoy)x) = G(xoyx) = -\sum_{i=0}^{i=m} x^i[x,y]\sum_{j=0}^{j=m} x^{j+1} = -\sum_{i=0}^{i=m} x^i[x,y]\sum_{j=0}^{j=m} x^{j+1}. \tag{18}$$

By definition of generalized derivation, we have $G(xoyx) = G(xoy)x + (xoy)d(x)$ Combining 18 and property (iv), we obtain

$$-\sum_{i=0}^{i=m} x^i[x,y]\sum_{j=0}^{j=m} x^{j+1} = -\sum_{i=0}^{i=m} x^i[x,y]\sum_{j=0}^{j=m} x^{j+1} + (xoy)d(x).$$

This implies that $(xoy)d(x) = 0$. Use the same way as above to get the required result.

Proof of Theorem 2.3. Assume the condition (v) of (p_2) as follows

$$G[x,y] = f(y)(xoy)g(y). \tag{19}$$

Replacing x by xy in (19), we get

$$G[xy,y] = G[x,y]y = f(y)(xoy)yg(y), \forall x,y \in N. \tag{20}$$

By definition of generalized derivation, we have

$$G([x,y]y) = G([x,y])y + [x,y]d(y). \tag{21}$$

Substituting the values of (19) and (20) in (21), we get

$$\sum_{r=0}^{r=p} y^r(xoy)\sum_{s=0}^{s=q} y^{s+1} = \sum_{r=0}^{r=p} y^r(xoy)\sum_{s=0}^{s=q} y^{s+1} + [x.y]d(y).$$

This implies that

$$[x,y]d(y) = 0. \tag{22}$$

Putting x by xz in (22), we find that

$$[xz, y]d(y) = [x, y]zd(y) = 0, \forall x, y, z \in N.$$

This implies that

$$[x, y]zd(y) = 0, \forall x, y, z \in N. \tag{23}$$

By an application of primeness of N, the relation (23) gives that for each $y \in N$

$$d(y) = 0 \textit{ or } y \in Z(N). \tag{24}$$

Taking $y \in Z(N)$, then

$$yx = xy, \forall x \in N. \tag{25}$$

Taking derivation both sides in (25), we have $d(xy) = d(yx)$.

This implies that

$$d(y)x + yd(x) = d(x)y + xd(y),$$

but $y \in Z(N)$, then $d(y)x = xd(y), \forall x \in N$ or $d(y) \in Z(N)$.

Hence, (24) implies that for all $y \in Z(N), d(y) \in Z(N)$, that is, $d(N) \subseteq Z(N)$.

In view of Fact 2.2, we obtain N is a commutative ring.

Further, if condition (vi) of (p_2) holds, then putting xy for x in (vi), we can similarly find that

$$G([x, y]y) = G([xy, y]) = -\sum_{r=0}^{r=p} x^r(xoy)\sum_{s=0}^{s=q} x^{s+1} = -\sum_{r=0}^{r=p} x^r(xoy)\sum_{s=0}^{s=q} x^{s+1}. \tag{26}$$

By definition of generalized derivation, we have $G([x, y]y) = G([x, y])y + [x, y]d(y)$.

Combining (26) and property (vi), we obtain

$$-\sum_{r=0}^{r=p} x^r(xoy)\sum_{s=0}^{s=q} x^{s+1} = -\sum_{r=0}^{r=p} x^r(xoy)\sum_{s=0}^{s=q} x^{s+1} + [x.y]d(y).$$

This implies that $[x,y]d(y) = 0$. Using the same way as above to obtain the requited result.

Proof of Theorem 2.4. Assume the condition (vii) of (p_3) as follows

$$G(xoy) = f(y)[x,y]g(y). \tag{27}$$

Replacing x by xy in (27), we get

$$G(xyoy) = G(xoy)y = f(y)[x,y]yg(y), \ \forall\, x,y \in N. \tag{28}$$

By definition of generalized derivation, we have

$$G((xoy)y) = G(xoy)y + (xoy)d(y). \tag{29}$$

Putting the values of (27) and (28) in (29), we find that

$$\sum_{r=0}^{r=p} x^r[x,y]\sum_{s=0}^{s=q} x^{s+1} = \sum_{r=0}^{r=p} x^r[x,y]\sum_{s=0}^{s=q} x^{s+1} + (xoy)d(y).$$

This implies that

$$(xoy)d(y) = 0 \;\forall\, x,y \in N$$

or

$$yxd(y) = -xyd(y) \;\forall\, x,y \in N. \tag{30}$$

Putting x by zx in (30), we get

$$yzxd(y) = -zxyd(y) = (-z)(-yxd(y)) = (-z)(-y)xd(y), \ \forall\, x,y,z \in N.$$

This gives

$$(yz - (-z)(-y))xd(y) = 0 \;\forall\, x,y,z \in N. \tag{31}$$

Putting y by $-y$ in (31), we get $(-yz + zy)xd(-y) = 0$, which implies

$$-[z,y]xd(y) = [z,y]xd(-y) = 0, \ \forall\, x,y,z \in N. \tag{32}$$

By an application of primeness of N, the relation (32) gives that for each $y \in N$

$$d(y) = 0 \text{ or } y \in Z(N). \tag{33}$$

It is easily seen that, if $y \in Z(N)$, then $d(y) \in Z(N)$. Thus relation (33) implies that $\forall y \in N,\ d(y) \in Z(N)$ or $d(N) \subseteq Z(N)$. In view of the Fact 2.2 N is a commutative ring.

Now, taking the property (viii) of (p_3) holds, then putting xy for x in (viii), we can similarly obtain

$$G((xoy)y) = G(xyoy) = \sum_{r=0}^{r=p} x^r [x, y] \sum_{s=0}^{s=q} x^{s+1} = \sum_{r=0}^{r=p} x^r [x, y] \sum_{s=0}^{s=q} x^{s+1}. \qquad (34)$$

This implies that $(xoy)d(y) = 0$. The rest of the proof is same as earlier condition (vii).

2.1 Remark

It is worth highlighting that for some particular value of positive integers m, n, p and q in Theorems 2.1, 2.2, 2.3 and 2.4 reduce to main results in Boua and Oukhtite (2011), Gölbaşi and Koç (2009) and extend results in Khan and Madugu (2017) for generalized derivations.This uniformity approves that our results are effective.

3. Results on the Prime Graph $P(N)$

Combine the Facts 2.3 and 2.4, to obtain the following corollaries of our results on the prime graph $P(N)$.

Result 3.1. Let N be a prime near-ring. If the prime graph $P(N)$ is a star and there exist positive integers m, n such that N admits a generalized derivation G associated with a non-zero derivation d satisfying properties $(p), (p_1)$ in Theorems 2.1 and 2.2 then the zero-divisor graph of N is a sub graph of $P(N)$.

Result 3.2. Let N be a prime near-ring. If the prime graph $P(N)$ is a star and there exist positive integers such that N admits a generalized derivation G associated with a non-zero derivation d satisfying properties $(p_2), (p_3)$ in Theorems 2.3 and 2.4 then the zero-divisor graph of N is a sub graph of $P(N)$.

4. Counterexamples

The following examples exhibit that the primeness hypothesis in Theorems 2.1-2.4 is needed even in the case of arbitrary rings.

Example 4.1. Let R be a non-commutative ring and

$$N = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} \;\middle|\; \forall \; x, y \in R \right\}.$$

Define a map $d : N \longrightarrow N$ by

$$d \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

one can easily see that d is non-zero derivation on N.

Let

$$B = \begin{pmatrix} 0 & 0 & \beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$\beta \neq 0$. Then $BNB = \{0\}$, which shows that N is not prime. Taking

$$G \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

then G is a generalized derivation on N that satisfies the property

$$G([A, B]) = A \circ B$$

or

$$G(A \circ B) = [A, B]$$

for all $A, B \in N$, and N is a non-commutative ring.

Example 4.2. Let R be a non-commutative ring and

$$N = \left\{ \begin{bmatrix} \alpha & \beta \\ 0 & 0 \end{bmatrix} \;\middle|\; \alpha, \beta \in R \right\}.$$

Define a map $d : N \longrightarrow N$ by

$$d\Big(\begin{bmatrix} \alpha & \beta \\ 0 & 0 \end{bmatrix}\Big) = \begin{bmatrix} 0 & \beta \\ 0 & 0 \end{bmatrix},$$

one can easily see that d is non-zero derivation on N.

Let

$$B = \begin{bmatrix} 0 & \beta \\ 0 & 0 \end{bmatrix},$$

$\beta \neq 0$. Then $BNB = \{0\}$, which shows that N is not prime. Taking

$$G\Big(\begin{bmatrix} \alpha & \beta \\ 0 & 0 \end{bmatrix}\Big) = \begin{bmatrix} 0 & \beta \\ 0 & 0 \end{bmatrix},$$

then G is a generalized derivation on N that satisfies the property

$$G([A, B]) = A[A, B]A$$

for all $A, B \in N$, and N is a non-commutative ring. Also, an alternate example can be found in Shang (2015).

5. Concluding Remarks

Author concludes this paper by considering some reservations for further research. The properties $(p), (p_1), (p_2)$ and (p_3) are assumed to be held for all $x, y \in N$.

Does Theorem 2.1 or Theorem 2.2 or Theorems 2.3 or Theorem 2.4 still true if above conditions hold for only $x, y \in S \subseteq N$, where S is suitable type of non-zero ideal of N?

The commutativity of torsion-free near-rings is additional fascinating forthcoming research effort.

6. Open Problems

One can look more general constraints on the some type of derivations and the constraints such as commutativity of torsion-free near-rings would be interesting.

6.1. Let N be a prime near-ring. If N admits a non-zero skew-derivation (skew-generalized derivation) d such that $d(xy - yx) = \alpha x^p(xy + yx)x^q$ or $d(xy-yx) = \alpha y^p(xy-yx)y^q$, for any $x, y \in N$ and α, p, q are positive integers, then N is a commutative ring.

6.2. Let N be a prime near-ring and there exist nonnegative integers β, p, q. If N admits a non-zero multiplicative derivation (multiplicative generalized derivation) δ such that $\delta(xy - yx) = \beta x^p(xy + yx)x^q \in Z(N)$ or $\delta(xy - yx) = \beta y^p(xy - yx)y^q \in Z(N)$, for any $x, y \in N$, then N is a commutative ring.

6.3. The conditions $(p) - (p_3)$ are assumed to be held for all $x, y \in N$. Does Theorem (2.1)or (2.2)or (2.3) or (2.4) true if these conditions held for only $x, y \in S \subseteq N$, where S is a semigroup ideal of prime near-ring N?

Acknowledgement

The author is grateful to Research Grant (UMYU/UBR/BC/003) for providing the financial assistance of this work. The grant number is Tetfund (TETUND/ESS.D/10./LR/PROFILE/RES), Umaru Musa Yar'adua University, Katsina- Nigeria. Author also thanks Mr. Tasiu Abdullahi Yusuf, Assistant Lecturer in the Department of Mathematics and Computer Science, for all the effort he made during the compilation for the paper.

References

Anderson, D. and Naseer, M. (1993). Beck's coloring of a commutative ring. *Journal of Algebra*, 159(2):500–514.

Anderson, D. F. and Livingston, P. S. (1999). The zero-divisor graph of a commutative ring. *Journal of Algebra*, 217(2):434–447.

Beck, I. (1988). Coloring of commutative rings. *Journal of Algebra*, 116(1):208–226.

Beidar, K., Fong, Y., and Wang, X. (1996). Posner and herstein theorems for derivations of 3-prime near-rings. *Communications in Algebra*, 24(5):1581–1589.

Bell, H. E. and Mason, G. (1987). On derivations in near-rings. 137:31–35. https://doi.org/10.1016/S0304-0208(08)72283-7.

Bhavanari, S., Kuncham, S. P., and Kedukodi, B. S. (2010). Graph of a nearring with respect to an ideal. *Communications in Algebra*, 38(5):1957–1967.

Boua, A. and Oukhtite, L. (2011). Generalized derivations and commutativity of prime near-rings. *Journal of Advanced Research in Pure Mathematics*, 3(2):120–124.

Daif, M. N. and Bell, H. E. (1992). Remarks on derivations on semiprime rings. *International Journal of Mathematics and Mathematical Sciences*, 15(1):205–206.

Gölbaşi, Ö. and Koç, E. (2009). Notes on commutativity of prime rings with generalized derivation. *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics*, 58(2):39–46.

Groenewald, N. (1991). Different prime ideals in near-rings. *Communications in Algebra*, 19(10):2667–2675.

Khan, M. A. and Madugu, A. (2017). Some commutativity theorems for prime near-rings involving derivations. *Journal of Advances in Mathematics and Computer Science*, 25:1–9.

Redmond, S. P. (2004). Structure in the zero-divisor graph of a noncommutative ring. *Houston Journal of Mathematics*, 30(2):345–355.

Robinson, D. F. and Foulds, L. R. (1980). *Digraphs: Theory and Techniques*. Gordon & Breach Science Pub.

Shang, Y. (2015). A note on the commutativity of prime near-rings. In *Algebra Colloquium*, volume 22, pages 361–366. World Scientific. https://doi.org/10.1142/S1005386715000310.

Wang, X. K. (1994). Derivations in prime near-rings. In *Proceedings of the American Mathematical Society*, volume 121, pages 361–366. https://doi.org/10.2307/2160409.